Indian Statistical Institute

Back Paper Examination 2014-2015

B.Math Third Year

Complex Analysis

Time : 3 Hours   Date : 05.01.2015   Maximum Marks : 100   Instructor : Jaydeb Sarkar

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(i) Answer all questions. (ii) $B_r(z_0) := \{z \in \mathbb{C} : |z - z_0| < r\}$. (iii) $C_r(z_0) := \{z \in \mathbb{C} : |z - z_0| = r\}$. (iv) $U \subseteq \mathbb{C}$ open. (v) $\mathrm{Hol}(U) = \{f : U \to \mathbb{C}$ holonorphic $\}$.

Q1. (15 marks) State and prove Rouche's theorem.

Q2. (15 marks) "The function $\frac{\sin z}{z^2}$ has no primitive in $\mathbb{C} \setminus \{0\}$." - True or false (with justification)?

Q3. (15 marks) Let $f \in \mathrm{Hol}(B_1(0))$, $f(0) = 0$ and $f(B_1(0)) \subseteq \overline{B_1(0)}$. Prove that the series

$$\sum_{n=0}^{\infty} f(z^n)$$

converges uniformly on compact subsets of $B_1(0)$.

Q4. (15 marks) Let $f \in \mathrm{Hol}(\mathbb{C})$. Suppose that there are positive real numbers a, b, and natural number n such that

$$|f(z)| \leq a + b|z|^n,$$

for all $z \in \mathbb{C}$. Prove that f is a polynomial.

Q5. (15 marks) Let f be a continuous function on U and $e^{f(z)} = z$ for all $z \in U$. Prove that $f \in \mathrm{Hol}(U)$. Compute f'.

Q6. (10+15 = 25 marks) Let $\mathbb{H} \subseteq \mathbb{C}$ be the open upper half plane.

(i) Prove that $B_1(0)$ and $\mathbb{H}$ are biholomorphically equivalent.

(ii) Let $\phi_1, \phi_2 : \mathbb{H} \to B_1(0)$ be a pair of biholomorphic maps. How are ϕ_1 and ϕ_2 related to each other?